بنام خدا

دانشکده مهندسی مکانیک تربیت مدرس	ترم دوم 99-00	استاد درس : کریمیان

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انتهای دروس 5 تا 8 کتاب فارلو

درس شماره 5

2. Show $\int_0^1 \sin(\pi m x)\sin(\pi n x)\,dx = \begin{cases} 0 & m \neq n \\ 1/2 & m = n \end{cases}$

HINT Use the identity

$$\sin(mx)\sin(nx) = \frac{1}{2}[\cos(m-n)x - \cos(m+n)x]$$

4. what is the solution to the IBVP

$$\begin{aligned} &\text{PDE} \quad u_t = u_{xx} \quad 0 < x < 1 \\ &\text{BCs} \quad \begin{cases} u(0,t) = 0 \\ u(1,t) = 0 \end{cases} \quad 0 < t < \infty \\ &\text{IC} \quad u(x,0) = 1 \quad 0 \leq x \leq 1 \end{aligned}$$

5. What is the solution to problem 4 if the IC is changed to

$$u(x,0) = \sin(2\pi x) + \frac{1}{3}\sin(4\pi x) + \frac{1}{5}\sin(6\pi x)$$

6. What would be the solution to problem 4 if the IC were

$$u(x,0) = x - x^2 \quad 0 < x < 1$$

درس شماره 6

1. Solve the initial-boundary-value problem

$$\begin{aligned} &\text{PDE} \quad u_t = \alpha^2 u_{xx} \quad 0 < x < 1 \\ &\text{BCs} \quad \begin{cases} u(0,t) = 1 \\ u_x(1,t) + hu(1,t) = 1 \end{cases} \quad 0 < t < \infty \\ &\text{IC} \quad u(x,0) = \sin(\pi x) + x \quad 0 \leq x \leq 1 \end{aligned}$$

2. Transform

$$\text{PDE} \quad u_t = u_{xx} \quad 0 < x < 1$$

$$\text{BCs} \quad \begin{cases} u(0,t) = 0 \\ u(1,t) = 1 \end{cases} \quad 0 < t < \infty$$

$$\text{IC} \quad u(x,0) = x^2 \quad 0 \leq x \leq 1$$

to zero BCs and solve the new problem. What will the solution to this problem look like for different values of time? Does the solution agree with your intuition? What is the steady-state solution? What does the transient solution look like?

3. Transform

$$\text{PDE} \quad u_t = u_{xx} \quad 0 < x < 1$$

$$\text{BCs} \quad \begin{cases} u_x(0,t) = 0 \\ u_x(1,t) + hu(1,t) = 1 \end{cases} \quad 0 < t < \infty$$

$$\text{IC} \quad u(x,0) = \sin(\pi x) \quad 0 \leq x \leq 1$$

into a new problem with zero BCs; is the new PDE homogeneous?

درس شماره 7

1. Solve the following heat-flow problem:

$$\text{PDE} \quad u_t = u_{xx} \quad 0 < x < 1 \quad 0 < t < \infty$$

$$\text{BCs} \quad \begin{cases} u(0,t) = 0 \\ u_x(1,t) = 0 \end{cases} \quad 0 < t < \infty$$

$$\text{IC} \quad u(x,0) = x \quad 0 \leq x \leq 1$$

by separation of variables. Does your solution agree with your intuition? What is the steady-state solution?

3. Solve the following problem with insulated boundaries:

$$\text{PDE} \quad u_t = u_{xx} \quad 0 < x < 1 \quad 0 < t < \infty$$

$$\text{BCs} \quad \begin{cases} u_x(0,t) = 0 \\ u_x(1,t) = 0 \end{cases} \quad 0 < t < \infty$$

$$\text{IC} \quad u(x,0) = x \quad 0 \leq x \leq 1$$

Does your solution agree with your interpretation of the problem? What is the steady-state solution?; does this make sense?

درس شماره 8

1. **Solve the diffusion problem**

$$\text{PDE} \quad u_t = u_{xx} - u_x \qquad 0 < x < 1 \qquad 0 < t < \infty$$

$$\text{BCs} \quad \begin{cases} u(0,t) = 0 \\ u(1,t) = 0 \end{cases} \qquad 0 < t < \infty$$

$$\text{IC} \quad u(x,0) = e^{x/2} \qquad 0 \le x \le 1$$

by transforming it into an easier problem. What does the solution look like? We could interpret this problem as describing the concentration $u(x,t)$ in a moving medium (moving from left to right with velocity $v = 1$) where the concentration at the *ends* of the medium are kept at zero (by some filtering device) and the *initial concentration* is $e^{x/2}$. Does your solution agree with this interpretation?

2. Solve the problem

$$\text{PDE} \quad u_t = u_{xx} - u + x \qquad 0 < x < 1 \qquad 0 < t < \infty$$

$$\text{BCs} \quad \begin{cases} u(0,t) = 0 \\ u(1,t) = 1 \end{cases} \qquad 0 < t < \infty$$

$$\text{IC} \quad u(x,0) = 0 \qquad 0 \le x \le 1$$

by

(a) changing the nonhomogeneous BCs to homogeneous ones.
(b) transforming into a new equation without the term $-u$.
(c) solving the resulting problem.

3. Solve

$$\text{PDE} \quad u_t = u_{xx} - u \qquad 0 < x < 1 \qquad 0 < t < \infty$$

$$\text{BCs} \quad \begin{cases} u(0,t) = 0 \\ u(1,t) = 0 \end{cases} \qquad 0 < t < \infty$$

$$\text{IC} \quad u(x,0) = \sin(\pi x) \qquad 0 \le x \le 1$$

directly by separation of variables without making any preliminary transformation. Does your solution agree with the solution you would obtain if the transformation

$$u(x,t) = e^{-t}w(x,t)$$

were made in advance?

موفق باشید

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